1. Schroedinger's Time Independent Equation (STIE). 6/16/09 dsm

So, here we start; with what I think I know w/o having to work at it.
STIE for a single particle is:

$$-h^2/(8\mathrm{m}\pi^2)\,grad^2\,\Psi(r)+V(r)\Psi(r)=E\,\Psi(r) \qquad (1.1))$$

where: m is mass, V(r) is the potential, h is Planck's constant, grad is grad (understand), and E is energy (and a constant). Where we elect to lump everything on the left hand side (LHS) into the operator H (as in Hamiltonian), we can write for STIE:

$$H\Psi=E\Psi \qquad (1.2)$$

Further, we assume H is hermitian , $H^{+}=H$; thus E is real.

This equation is obviously in the form of an eigenfunction; that is, an operator (H), on a function Ψ , is equal to a constant (E) times the function Ψ . Now generally, a given, specific solution yields the following:

$$H\Psi_a=E_a\Psi_a \qquad (1.3)$$

Where we call the $\Psi_a's$ wavefunctions; and they constitute a set of functions that embody the following characteristics:

1. they are orthogonal, that is $\int \Psi_b^{*}\Psi_a dr=0\ for\ b\neq a$; (1.4)
2. they are complete, that is $\Psi=\sum_i a_i\Psi_i$ for any/all arbitrary Ψ satisfying (1.2) ;
3. they are closed, that is they include their limit points; and
4. they are normal (normalized to unity), that is $\int \Psi_a^{*}\Psi_a dr=1$ (1.5)

Which is to say, they constitute a normalized, linear, vector space; a Hilbert space.
Now, we immediately arabesque to the famous Dirac notation:

$\Psi_a=$|a> , $\int \Psi_b^{*}\Psi_a dr=$<b|a> , and since the |a>'s constitute a complete set:

$$\sum_i |i\rangle\langle i|=I \qquad (1.6)$$

Where I is the unity operator, (= 1); which means that it can be inserted anywhere w/o changing the results of a given calculation; we will be quite promiscuous, prodigious, and cavalier in throwing this operator around, as will be seen shortly.

Furthermore, we will take recourse in calculating the interaction due to an arbitrary perturbation, H', via the formulation:

$$\int \Psi_a^{*}H'\Psi_b dr=\langle a|H'|b\rangle \quad \text{not generally} = 0. \qquad (1.7)$$

Here endeth the lesson.